Understanding Risk Measurement Tools

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Abstract

Many methods exist for assessing and managing the risk of a portfolio. This article is about risk metrics and the ways investment consulting practitioners commonly apply these measurements to the portfolio selection and evaluation process. Two popular approaches to risk measurement and evaluation are compared: the mean-variance and time-state paradigms. The relative merits and uses of each are presented with the conclusion that the time-state paradigm, with a new metric, Total Assessed Risk (TaR, a variant of Value at Risk), is the more robust and meaningful risk metric for investment management consultants designing and overseeing investment portfolios.

Introduction

What are the "best" measures of risk to use when evaluating a manager or fund in the portfolio selection and maintenance process? Selecting an appropriate portfolio for a client is not merely a matter of evaluating historical returns and assuming that the past will repeat itself. Yes, investors care about historical returns (and the risks that were taken to achieve them) when considering how to invest their money, but *investing is a prospective science and art*, and we cannot know the future by merely looking in the rear view; though where we have been can give us some guidance as to where we might be going and how we might get there.

Given that investors tend to be risk averse (i.e., they are usually willing to pay some price to reduce uncertainty), it is necessary to carefully consider risk when evaluating past performance and investing prospectively on their behalf. What then is risk and how should we measure it?

What is Risk?

There are many types of investors. There are individuals and institutions (both we refer to as agents throughout this article) with a variety of liability funding needs, investment objectives, risk preferences/tolerances, constraints, experience, and current investment holdings. Each agent is therefore affected by and perceives uncertainty in uniquely personal ways. For example, an agent who is long a security may view the risk of upside potential very differently from the agent who is short the very same security. Therefore, let us define risk in investing as the probability of an adverse divergence from an agent's outcome or preferred income stream.

How Should We Measure Risk?

Enumerating, quantifying, and assigning probabilities to possible future events are essential to distinguishing between risk and uncertainty—in a mathematical sense. This can be done with first-order risk measurement models directly, while second-order models only measure risk indirectly.

A first-order model of risk, based on our definition from the previous section, would measure risk directly by specifying all possible income streams or outcomes achievable in the future and assigning probabilities to them. This first-order model is called a "time-state" paradigm of risk because it specifies the timing of particular events, which are called "states," as in "states of nature." A commonly used first-order model is value at risk (VaR) which measures the maximum dollar value of a portfolio that can be lost, over a given time frame, with a certain degree of confidence.

Second-order models of risk measurement are the most commonly used in the investment consulting industry (e.g. the mean-variance model of Markowitz). This approach, and in fact all second-order models, make important simplifying mathematical assumptions —that is what makes them "second-order" models. Using second-order statistical approaches is often desirable in finance because they have convenient mathematical properties and are easy to use. *However, these models do not directly measure first-order phenomenon, like risk.* First order models would be far more powerful and accurate tools to have at our disposal when considering risk in the portfolio selection and evaluation process (this does not imply that second order models are not useful and helpful; they are but one must understand the limits of their utility). In this article, we clarify when to use these second-order models, and how to use the first-order models.

Second Order or Mean-Variance Measures

The most popular measures of risk and risk-adjusted performance are derived from the Markowitz mean-variance paradigm for dealing with investment choice. This model advances the idea that investors have a utility function that depends on their perception of possible returns, probabilities of those possible returns, risk (variance of possible returns), and risk aversion, generally expressed as $U = E\left(r_p\right) - .005A\sigma_p^2$. In short, an investor's portfolio selection problem boils down to selecting a portfolio such that their utility is maximized. Mathematically, this is equivalent to maximizing the reward-to-variability ratio (i.e., the Sharpe ratio) for their entire portfolio.

If, in addition to assuming investors have a utility curve that is quadratic like the preceding description, we assume that there are no market frictions (e.g. brokerage costs, taxes, etc.), investors have homogeneous expectations about the future, investors can borrow and lend at the risk-less interest rate, we get the standard capital asset pricing model (CAPM) as formulated by Sharpe (1964) and Lintner (1965).

CAPM shows the relationship between the expected risk premium of a security and the expected risk premium of the most well diversified portfolio (e.g., the market portfolio) available. Mathematically this is expressed as $E\left(r_p\right) = r_f + \beta_p\left[E\left(r_M\right) - r_f\right]$. The risk of a security is defined by the security's beta, where $\beta_p = \frac{\sigma_{p,M}}{\sigma_M^2}$.

Researchers have learned how to make the CAPM more realistic by eliminating a lot of the market assumptions necessary to construct the CAPM. The two that cannot be eliminated—and maintain mathematically tractable results—are the assumptions about investors' expectations and preferences. These are important assumptions to eliminate though, as they are the ones that relate to people—our clients and their behavior in our real financial world.

Along with the CAPM came a host of risk adjusted performance measures from Jack Treynor (1966), William Sharpe (1966), and Michael Jensen (1968 and 1969). Their proposed measures, and variants, are still fashionable today. Table 1 shows the different risk adjusted return measures that have become common in practice. These seven measures when considered as a group can assist the consultant in answering the question of whether one portfolio (or security) is better than another for a particular investor and they can assist in separating basic ability from random luck in portfolio selection—provided you are prepared to accept that the investor has a mean-variance specified utility function (the Sortino measure and the upside potential ratio do not require these assumptions).

These measures rely on some extreme forms of market efficiency posited by the rational expectations theory as outlined by Muth (1961) and applied to financial markets. There are plenty of anomalies to market efficiency that are not well explained, however, that need not be recounted here (see Lamont and Thaler (2003) for a review)—especially when management fees and information costs are taken into consideration. In fact, these forms do not often evaluate what they are intended to measure (Leggio, 2003). But, if markets are efficient, <u>and</u> investors do have quadratic utility functions, then portfolio selection is simple: pick the portfolio that maximizes the Sharpe ratio.

Table 1 Mean-Variance Portfolio Selection Tools

Name	Formula	Description	When to Apply
Sharpe Ratio	$\left(\overline{r}_p - \overline{r}_f\right) / \sigma_p$	Divides average portfolio excess returns over the sample period by the standard deviation of returns over that period. This is also called the reward-to-variability (or volatility) ratio	Use when the potential investment is the agent's entire stock of liquid investment wealth.
Jensen's Measure (Portfolio Alpha)	$\alpha_p = \overline{r}_p - \left[\overline{r}_f + \beta_p\left(\overline{r}_M - \overline{r}_f\right)\right]$	This is the average return on a portfolio beyond that predicted by the CAPM.	Use when considering whether to add a security to an already well-diversified portfolio.
Information Ratio (or Appraisal Ratio)	$\alpha_p / \sigma\left(e_p\right)$	This is the alpha of a portfolio divided by the diversifiable, idiosyncratic, residual risk of the portfolio. The risk in the denominator is also called the "tracking error."	Use to measure the marginal contribution made to the Sharpe ratio by active managers.
Treynor's Measure	$\left(\overline{r}_p - \overline{r}_f\right) / \beta_p$	This is the excess return per unit of systematic risk (as measured by beta).	Use when choosing between two well-diversified portfolios to add to an already diversified portfolio.
M^2	$M^2 = r_{p*} - r_M$	This is calculated by imputing the return on a portfolio of the managed portfolio and a risk-free security so that the adjusted portfolio's volatility matches that of the market index.	Use in a similar manner as the Sharpe ratio: when the potential investment is the agent's entire stock of liquid investment wealth.
Sortino's Omega	$\omega = \left(\overline{r}_p - \overline{r}_f\right) / \varsigma^2$ where ς^2 is the semi-variance	This is used to take into consideration that investors do	Use when the potential investment is the

		not mind "happy surprises," but they do not like "unhappy surprises" where returns or values are below a certain threshold.	agent's entire stock of liquid investment wealth and the agent does not have a mean-variance utility curve.
Upside potential ratio	$UPR = \left(\overline{r}_p - \overline{r}_b\right) / \varsigma^2$	Instead of comparing the managed portfolio to the risk-free rate, it is compared to some appropriate benchmark, or a minimal acceptable return.	Use when considering adding a security to an already well diversified portfolio and the agent does not have a mean-variance utility curve.

With all of the preceding measures (in Table 1), a positive alpha is a necessary requirement for a security to be added to a portfolio. However, alpha is not a sufficient condition for superior performance, as the alpha might not be worth the cost (i.e, the risk).

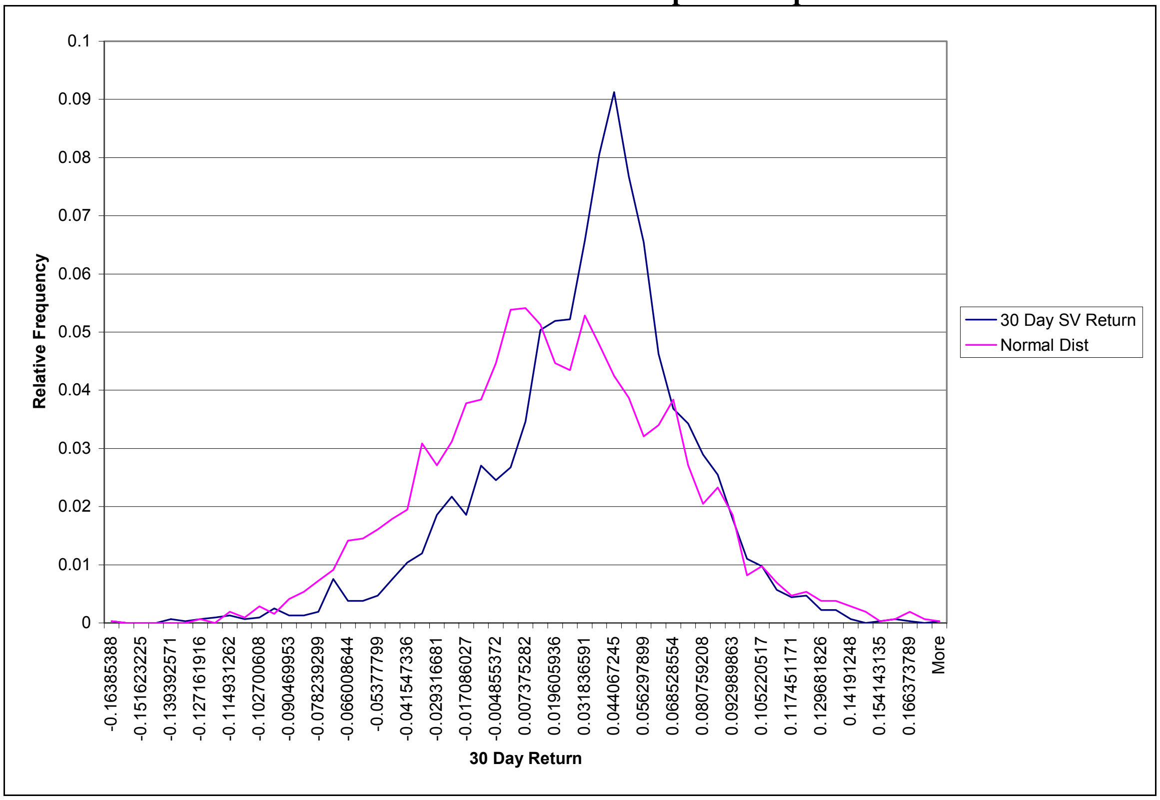
First Order or Distributional Risk Measures

As stated in the introduction, an alternative to the mean-variance paradigm—a second-order model—and its measures, is a time-state paradigm—a first-order model. The first-order model provides robustness and accuracy that second-order models simply cannot attain. The benefits of this additional accuracy are that it can provide the practitioner with better data for sharper, clearer decision making and potentially improved portfolio performance as will be shown later in this paper.

In this paradigm, the actual or potential distribution of returns is analyzed and different measures of risk are calculated. The most popular risk measurement is value-at-risk (VaR). VaR and its variants are now critical risk management tools, especially for financial organizations. It has become popular with financial institutions and regulators (e.g. Bank for International Settlements and the SEC) because it focuses on the losses (or adverse outcomes) that companies could make with a reasonable probability. Like all measures, VaR has certain limitations in its popular formulation.

In its classic formulation, VaR assumes that values or outcomes follow a normal distribution, which can be characterized by the mean and variance of the distribution; however, that assumption can be discarded and *any* distribution can be assumed for the outcomes making this more robust and realistic than the mean-variance paradigm which assumes and relies on a symmetrical distribution (e.g. see Figure 1 for the monthly return distribution for small cap value stocks).

Figure 1: 30 day Return Distribution for Small Cap Value Stocks (SV) from 1992 to 2004 compared to a hypothetical return distribution that is drawn from a normal distribution with the same mean and standard deviation of the empirical sequence.



The basic idea behind VaR is that risk is perceived as the likelihood, or probability, of specific undesirable outcomes—not just the standard deviation of possible outcomes. Thus, and here is the most important point we are making, any truly accurate risk measure should focus on the probability of losses (a first-order measure of risk) relative to some benchmark (i.e. a minimum acceptable return, a current value, a zero value, an expected value, etc.) and not merely the standard deviation of the possible outcomes (a second-order measure of risk). Why? Because that is the way real investors perceive their risk and it is the dollar value of their assets that determines what they can consume. A paramount role of the investment consultant serving the affluent investor is to guide or manage future consumption ability. Therefore, distributional measures of risk are far more intuitive for clients to grasp because they can reduce investment choice and, therefore, portfolio selection to first-order dollars and cents decisions.

Here is how VaR works. VaR uses the expected worst loss that occurs with a certain probability over a specified time horizon. Let V be the dollar value of an investment at the end of that time horizon. Define V* such that $P\left(V \le V^*\right) = \int_{-\infty}^{V^*} dF\left(V\right) = 1 - c$ where c denotes the confidence level and F(V) the cumulative distribution function of V. So, with probability 1-c, the value of the investment will be less than or equal to V*. The VaR is just this loss (V*) relative to some benchmark (e.g., the status quo, the expected value, etc.).

Using the distribution shown in Figure 2, the VaR for a $1,000,000 portfolio over 30 days, with 95% confidence is $100,269. This assumes that the normal distribution adequately captures the potential return distribution. Using TaR (to be explained below), the risk exposure is actually $70,529. Again, these numbers represent the largest dollar value loss of a portfolio over a given time horizon with given degree of certainty. In other words, if the VaR measure is accurate, losses greater than the VaR measure should occur less than a specified percentage of the time.

VaR risk measures however are not perfect and routinely fall short on one of two accounts (Artzner, 1999):

1. *Additivity Problem*: You can not measure the VaR of two securities and just add them to get the composite VaR. The reason for this is similar to why you cannot just add variances of securities to get the variance of the composite portfolio: there is correlation between the securities. VaR complicates somewhat because what is important in VaR calculations is the correlation of the losses below a certain threshold value, not an average across the entire distribution. This necessitates the use of computer simulations to determine portfolio VaRs.
2. *Distributional Problem*: The use of VaR depends critically on underlying return distributions. These are often assumed to follow a normal distribution, but there is ample evidence that returns do not follow normal distributions. A better solution, and one that can be implemented, is to use empirical observations to construct return distributions.

Managing portfolio risk is like squeezing a balloon: when you control one set of risks, another can arise spontaneously. Standard VaR models ignore the significance of this multi-dimensionality of risk; thereby misstating the true value-at-risk. We have developed a method called Total Assessed Risk (TaR) that is a generalization of VaR. What we do is estimate the distribution of returns from empirical observations on all possible portfolios of risks. VaR imposes a distribution on the returns, but we allow the distribution to be an emergent phenomenon from the actual returns. We simulate all possible combinations of securities in a portfolio to determine the true value-at-risk. This overcomes the problems cited above related to VaR (the additivity problem and the distributional problem).

TaR allows for easy communication of investment objectives with a client. To pick a portfolio along a standard capital market line requires knowledge of an investors “coefficient of risk aversion” which is a nebulous concept. The TaR method asks what the minimal acceptable return is, and with what degree of probability that must be met. This allows a robust method for structuring and managing portfolios for endowments, pension funds, and insurance companies who have actuarially determined rates of return that must be met for funding purposes. This is also a superior investment tool for communicating with high net worth clients and not so high net worth clients because the results align with the way investors look at their world and their money.

Comparing First Order and Second Order Risk Measurement Methods

In order to assess the validity of the above claims of robustness, we have conducted a simulation of investment strategies based on both the mean-variance criterion and on the distributional measures. We have compared the performance of the “tangency” portfolios selected using the Sharpe-Ratio maximization strategy to a TaR method. When using VaR and TaR risk measures for portfolio selection, all we need to know is the investor’s minimal acceptable return so that we can maximize the probability of meeting or exceeding that return. So, for illustrative purposes, we have simulated portfolios that attempt to maximize expected returns, subject to a number of different return probability constraints. The three return probability restrictions we illustrate in this paper are: to lose no more than 0% with 90% certainty of the outcome, and to lose no more than 5% per annum with both a 90% and an 80% certainty.

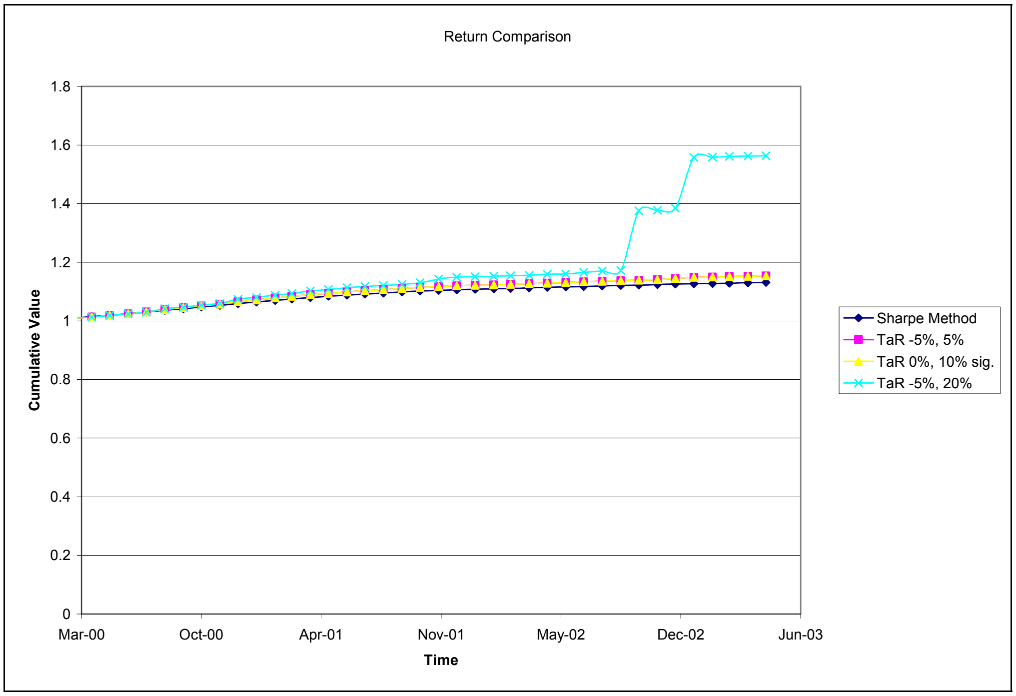
The investment simulation can be described as follows:

1. The investor may choose from a universe comprised of MSCI US Investable Market 2500 Index, the MSCI EAFE Index, and the Federal Funds rate (our proxy for the risk-free interest rate).
2. The portfolio is rebalanced every 30 days.
3. The simulation runs for over three years from February 2000 to March 2003. This covers the business cycle and swings in the market.
4. The investor invests according to one of two strategies:
    a. Sharpe ratio strategy: every period, the investor picks the portfolio that maximizes the Sharpe ratio, which is calculated using the previous 30 days of returns.
    b. TaR strategy: every period, the investor picks the portfolio that maximizes the expected return of the portfolio subject to the return probability constraints explained above. The returns probability distribution is calculated using a bootstrap method that uses the previous 30 day return sequence.

We ran the simulations for cases in which the investor could short securities and where the investor could not short securities. This simulation was run using data downloadable from MSCI and the U.S. Federal Reserve.

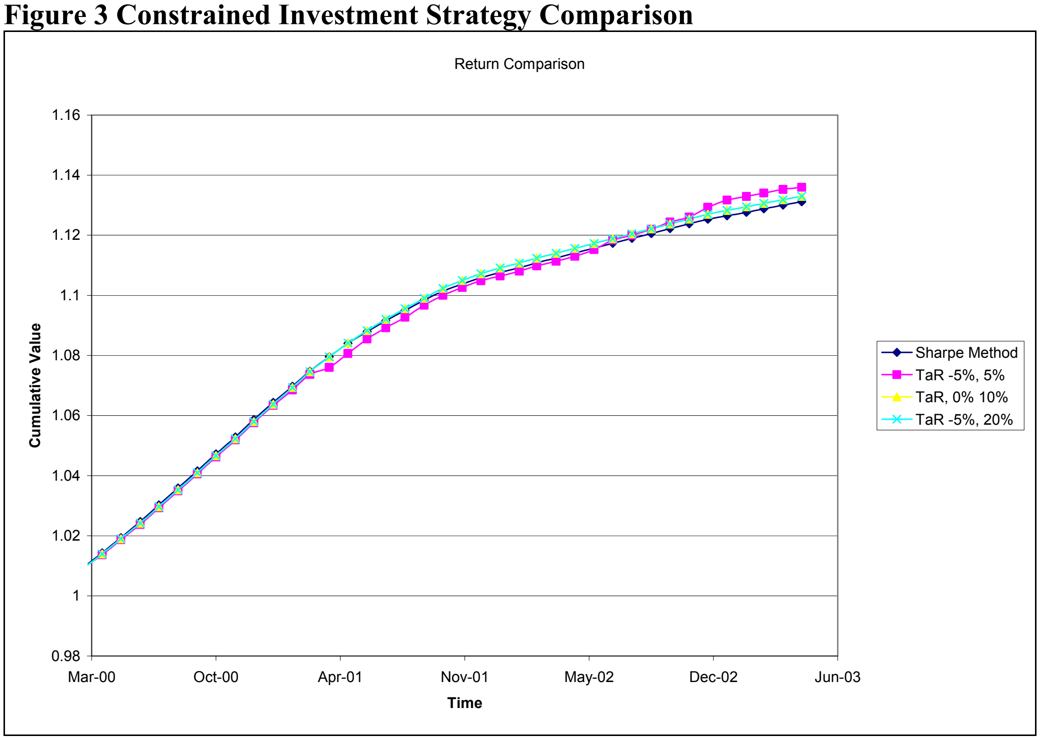
The following graph (Figure 2) shows the performance result of each strategy where the investor could short securities. The TaR method, where an investor is willing to lose no more than 5% per annum in portfolio value with 95% certainty outperformed the Sharpe-based method yielding a total gain in value of 56.33% versus 13.14%.

Figure 2 Unconstrained Investment Strategy Comparison



The following graph shows the performance result of each strategy where the investor could not short securities. The TaR method, where an investor is willing to lose no more than 5% per annum in portfolio value with 95% certainty outperformed the Sharpe-based method yielding a total gain in value of 13.60% versus 13.12%.

Figure 3 Constrained Investment Strategy Comparison



Regardless of short selling constraints, the TaR method—no matter what the return probability constraints imposed—dominated the Sharpe-based method in every period.

Conclusion

So, which risk measurement method for portfolio selection would a client prefer? We believe the TaR method. TaR is superior because its intuitiveness makes it easier for clients to become better informed about the rationale behind your portfolio guidance.

Which risk measurement method for portfolio selection and evaluation would investment consultants prefer? All things being equal, we believe TaR. TaR is a superior tool because its underlying—and less restrictive—assumptions measure risk more precisely and can be shown to produce superior investment results.

While mean-variance methods have been used for decades as the primary tool investment consultants have had in portfolio selection and evaluation, new methods, like TaR, will advance the state of the art. Second order risk measures will continue to have clinical usefulness, but in the years ahead, as new measurement tools become more commonplace and inexpensive to acquire, investment consultants will not be limited to only these traditional measures.

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